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RESEARCH PROJECT INITIATION

Date: October 11, 1973

Project Title: Solutions in Cosserat Surface Theory

Project No: E-23-606

Principal Investigator: Dr. Stephen L. Passman

Sponsor: National Science Foundation

Agreement Period: From October 1, 1973 Until March 31, 1976*

Type Agreement: Grant No. GK-40181

Amount: $32,100 NSF Funds (E-23-606)
14,206 GIT Contrib. (E-23-316)
$46,306 TOTAL

Reports Required:
Annual Technical Letter; Final Technical Summary

Sponsor Contact Person (s):

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National Science Foundation
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*All committments to be met by this date unless formal extension of the grant is obtained in advance. Basic work period (24 mos.) ends September 30, 1975.

Assigned to: School of Engineering Science & Mechanics

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RA-3 (6-71)

GEORGIA INSTITUTE OF TECHNOLOGY
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SPONSORED PROJECT TERMINATION

Posted 4/28 CH

Date: April 28, 1977

Project Title: Solutions in Cosserat Surface Theory

Project No: E-23-606

Project Director: Dr. Stephen L. Passman

Sponsor: National Science Foundation

Effective Termination Date: 8/31/76

Clearance of Accounting Charges: 8/31/76

Grant/Contract Closeout Actions Remaining: none

- _ Final Invoice and Closing Documents
- _ Final Fiscal Report
- _ Final Report of Inventions
- _ Govt. Property Inventory & Related Certificate
- _ Classified Material Certificate
- _ Other ____________

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ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N. W.
ATLANTA, GEORGIA 30332

October 10, 1974

Dr. Clifford Astill
Solid Mechanics Program
National Science Foundation
Washington, D.C. 20550

Dear Dr. Astill:

The following information is presented as a progress report on N.S.F. grant number GK-40181, "Solutions in Cosserat Surface Theory".

Considerable effort has been expended in searching for exact solutions for finite deformations within the theory, in the context of various types of constraints motivated by classical shell theory. In specific deformations which seem reasonable in view of physical experience, many of these constraints are satisfied trivially for all materials, thus rendering the usual simplification yielded by constitutive constraints irrelevant. Two new families of universal solutions have been found for the unconstrained elastic Cosserat surface, in which the Kirchhoff-Love hypotheses are relaxed. These families correspond roughly to shearing of the surfaces by tractions tangent to the faces of the surfaces. Although tractions of this nature do occur in engineering situations, I find no reference in the literature of shell theory to problems of this nature. This work is currently being typed for submission. In the course of this investigation an explanation, somewhat more specific than previously published, has been found for physical components of constraint forces in the Cosserat theory in terms of constraint forces in non-polar elasticity theory. This is presently being prepared for publication.

Note should be also made of my work in a related area. A method for classifying work in oriented materials is by the dimensions of the spaces involved: n, the embedding space; m, the intrinsic space; p, the director space; and q, the number of components. Thus a Cosserat surface would be described by a surface ($m = 2$) of three-dimensional vectors ($p = 3$), embedded in three-dimensional space ($n = 2$), where the constitutive equations describe one component ($q = 1$). It appears that a formally similar theory with $n = 3$, $m = 3$, $p = 1$, q an arbitrary integer, would, with proper constitutive assumptions, describe a mixture of granular media or, in physical terms, a "soil". Some effort has been expended in considering this case.

A paper on the balance laws was given at the Conference on Theoretical Rheology at Cambridge, U.K. in September, 1974, to be published in the Proceedings of this conference. A paper on the constitutive theory was presented at the Seminar on Mixtures and Structured Continua in Udine, Italy, in June, 1974. This work is being prepared for publication.

Reprints of the works mentioned will be forwarded upon receipt.

Respectfully,

S. L. Passman
Assistant Professor

vc

E-19-646

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

January 15, 1976

Dr. Clifford Astill
Solid Mechanics Program
National Science Foundation
Washington, D. C. 20550

Dear Dr. Astill:

The following information is presented as a progress report on N.S.F. Grant Number GK-40181, "Solutions in Cosserat Surface Theory":

As explained in the last progress report, research under the auspices of this Grant is being conducted on a wider range of 'structured', 'oriented', or 'Cosserat' continua than was anticipated in the proposal. This is because all such theories are formally quite similar. It often occurs, however, that the nature and structure of solutions which are uninteresting in the context of one particular theory might be quite interesting, perhaps providing new physical insight or explaining mathematically some observed phenomena, in the context of another particular theory.

In the area of Cosserat surface theory, a paper, "An Exact Solution in Cosserat Plate Theory," has been completed and will be presented at the Eighth Southeastern Conference on Theoretical and Applied Mechanics. Papers presented at this meeting are published in its Developments in Theoretical and Applied Mechanics. In addition, an invited lecture entitled, "The Inertia of a Cosserat Surfact," was presented at the Twelfth Annual Meeting of the Society of Engineering Science, in Austin, Texas, October, 1975. An abstract of this lecture is given on page 1051 of the Proceedings of this meeting. Research on inertia continues but a paper on this subject has not yet been submitted for publication.

In the area of granular media, the paper, "Balance Laws for Mixtures of Granular Materials," was presented at the Conference on Theoretical Rheology of the British Society of Rheology in September, 1974. This work appears on pages 169-185 of Theoretical Rheology, Applied Science Publishers, 1975. Copies are enclosed. A further work, "Mixtures of Granular Materials," which includes a constitutive theory, has been submitted for publication. Invited lectures on this subject matter will be given at Cornell and Tulane Universities in 1976.

Respectfully,

S. L. Passman
Associate Professor

SLP:pm

Enclosures

11

Balance Laws for Mixtures of Granular Materials

S. L. PASSMAN

ABSTRACT

A model of a dry granular medium as a structured continuum with a one-dimensional director has been presented in the literature. Physical phenomena of interest involve mixtures of several granular media, or a mixture of a granular medium with a fluid, including the possibility of chemical reaction. General and exact balance laws for these situations are formulated, and some of their implications are discussed.

11.1. INTRODUCTION

The construction of mathematical models for various phenomena in soil mechanics is of obvious interest. Many of these phenomena fall within the regions of applicability of the classical theories of elasticity, plasticity, viscosity, etc., grouped here under the general term ‘continuum mechanics’. Other phenomena in soil mechanics are somewhat more complex, their study requiring theories which allow coupling, sometimes non-linear, among several theories which classically have been studied separately.

One class of problems which have not been dealt with in a completely successful manner within the context of the classical theories is that class where the granular nature of the soil is important, *i.e.* manifests itself directly, yet the grains are too small or too numerous to be dealt with individually in the mathematical theory.

Inherent in the classical theories of continuum mechanics is an assumption that the material is, in some sense, smoothly distributed in space. This assumption is basic in that its violation would preclude some of the analytic operations required to arrive at the differential equations which govern these phenomena. It would appear, then, that to forgo this assumption in order to describe the granular structure of these would introduce considerable analytic difficulty.

An approach used in the past to describe materials in which the structure of the individual elements of the material affects the gross behaviour of the material in some way, but for which the scale of the interesting phenomena is considerably larger than the scale of the structure, is to retain the continuity assumption, but to assume that each material element, rather than being a point, has some more complicated structure. An example of such a model with foundations early in this century is the theory of liquid crystals,† where a fluid-like body, each element of which is a vector, is considered. In some sense, the classical theory of rods of Bernoulli and Euler is also of this nature. More recent theories of this type are a shell theory of E. and F. Cosserat, and modern theories for the motion of red blood cells.

An ingenious theory of this type of structured continuum was presented by Goodman and Cowin (1971, 1972) and subsequently developed by Lee *et al.* (1974) and Jenkins (1975). There, a model of a granular medium consisting of spherical grains with no material in the interstices is established. A novel part of the analysis is the inclusion of a scalar-valued 'director' which measures the proportion of the volume occupied by the medium which actually contains grains of the material. (This quantity, when subtracted from 1, would then yield what is usually called the 'porosity'.) A set of constitutive equations is assumed which is valid only for a flowing medium and for limiting static equilibrium. For these situations the theory is moderately successful. It predicts the Mohr–Coulomb criterion, and solutions to problems are similar to observed phenomena for such media, predicting the existence, for example, of plug flow.

In view of their nature, it would appear that a mixture theory for continua of this type, possibly including the special case when one or more components might be non-polar fluids, could be a reasonable model for a soil. Such a theory, just as any

†A bibliography of some of the important literature on this subject has been given by Leslie (1968).

reasonably formulated theory for mixtures, would be quite complicated in its full generality. Herein, we consider a simple case of a mixture theory for such materials. We consider n constituents, each of the same nature as the material of Goodman and Cowin. We present only general and exact balance laws. Constitutive equations appropriate to these balance laws and modelling a type of soil are given by Passman (1974*b*).

11.2. NOTATION

The notation used here is essentially the direct vector-dyadic notation of Gibbs and Wilson (1909) with slight modifications first introduced into the mechanics literature by Noll.

We avoid a detailed explanation of the notation, referring the reader rather to the article of Truesdell and Noll (1965), where such an explanation is given. Essentially, the difference between this notation and that of Gibbs is the convention that tensors of the second order, rather than being considered as independent algebraic entities, are thought of as linear transformations on a vector space. Thus, the well-known theorem of Cauchy on the existence of the stress tensor is expressed in the following fashion. Let $\mathbf{T}$ be the stress tensor at a point, $\mathbf{n}$ a unit normal to a plane through the point, and $\mathbf{t}$ the stress vector on that plane. Then

$$\mathbf{t} = \mathbf{T}\mathbf{n}$$

The same equation written in Gibbs notation is

$$\mathbf{t} = \mathbf{n} \,.\, \mathbf{T}$$

and the same equation in terms of Cartesian index notation would be

$$t_i = T_{ij} n_j$$

To clarify the notation somewhat, we write a few formulae in our present notation and in Cartesian index notation. The tensor product of two vectors is given by

$$\mathbf{A} = \mathbf{a} \times \mathbf{b} \quad \text{or} \quad A_{ij} = a_i b_j$$

The wedge, or outer product of two vectors, is given by

$$\mathbf{a} \wedge \mathbf{b} = \mathbf{a} \times \mathbf{b} - \mathbf{b} \times \mathbf{a}$$

This product is a skew-symmetric tensor with the same independent components as the vector product between **a** and **b**. Unlike the vector product, however, its components transform as components of a tensor, even under a change from right-handed to left-handed co-ordinates.

For the gradient of a vector

$$\mathbf{G} = \operatorname{grad} \mathbf{a}, \quad \mathbf{G}_{ij} = a_{i,j}$$

where the comma denotes covariant differentiation in the case of general co-ordinates.

11.3. PRELIMINARIES

The concepts of body, force and motion are assumed known to the reader, and are not explained here.

A sequence of bodies $\underset{a}{\mathscr{B}}$, $a = 1, 2, \ldots, h$, is considered. A fixed reference configuration is chosen for each body, and in this configuration $\underset{a}{\mathbf{X}}$ is the place occupied by a particle of $\underset{a}{\mathscr{B}}$. The motion of $\underset{a}{\mathscr{B}}$ is the smooth mapping

$$\underset{a}{\mathbf{x}} = \underset{a}{\mathbf{x}}(\mathbf{X}, t), \quad t \in [-\infty, \infty] \tag{3.1}$$

of $\underset{a}{\mathscr{B}}$ on to a region of three-dimensional Euclidean space $\mathscr{E}$. In general, a backward prime is used to denote a time derivative with $\underset{a}{\mathbf{X}}$ held fixed. Thus, the velocity and acceleration of constituent a are given by

$$\begin{aligned} \underset{a}{\dot{\mathbf{x}}} &= \partial_t \underset{a}{\mathbf{x}}(\underset{a}{\mathbf{X}}, t) = \underset{a}{\dot{\mathbf{x}}}(\mathbf{x}, t) \\ \underset{a}{\ddot{\mathbf{x}}} &= \partial_t^2 \underset{a}{\mathbf{x}}(\underset{a}{\mathbf{X}}, t) = \underset{a}{\ddot{\mathbf{x}}}(\mathbf{x}, t) \end{aligned} \tag{3.2}$$

where the second forms follow by the assumed smoothness of (3.1). Often the term 'peculiar . . .' will be used in place of '. . . of constituent a'. Thus, $\underset{a}{\dot{\mathbf{x}}}$ is the peculiar velocity of the ath constituent. It is assumed that, for each t, there is a region of $\mathscr{E}$ each point of which is occupied simultaneously by particles for each $\underset{a}{\mathscr{B}}$ Henceforth, each formula written will be assumed to hold in subregions or at points of this region.

Assume that each $\underset{a}{\mathscr{B}}$ has a mass, which is measured on $\underset{a}{\mathscr{B}}$, absolutely continuous with respect to volume on each configuration of $\underset{a}{\mathscr{B}}$ Then a mass density $\underset{a}{\rho}$ exists. Physically, $\underset{a}{\rho}$ represents the mass per unit volume of space. Likewise, it is assumed that for each configuration of $\underset{a}{\mathscr{B}}$ there is a distributed volume, absolutely continuous with respect to volume. Then a volume distribution function $\underset{a}{\nu}$ exists and satisfies the inequality

$$0 \leqslant \underset{a}{\nu} \leqslant 1$$

The distributed mass density $\underset{a}{\gamma}$ is defined by

$$\underset{a}{\rho} = \underset{a}{\gamma}\underset{a}{\nu}$$

Physically, $\underset{a}{\nu}$ models the proportion of the total volume occupied by constituent a and $\underset{a}{\gamma}$ is the mass of constituent a per unit volume occupied by constituent a.

The usual mechanical and thermodynamic fields are defined on each $\underset{a}{\mathscr{B}}$ These fields are:

$\underset{a}{\mathbf{b}}$, body force

$\underset{a}{s}$, body heating

$\underset{a}{\mathbf{T}}$, stress

$\underset{a}{\mathbf{q}}$, heat flux

$\underset{a}{\epsilon}$, internal energy

Each $\underset{a}{\mathcal{B}}$ is allowed to interact with every other body $\underset{a}{\mathcal{B}}$ pointwise in the sense that the bodies can exchange mass (such an effect might arise from a chemical reaction), linear and angular moments (interaction forces and couples), heat and energy. These interactions are expressed by the following 'growth' terms:

$\overset{+}{\underset{a}{c}}$, growth of mass

$\overset{+}{\underset{a}{\mathbf{m}}}$, growth of linear momentum

$\overset{+}{\underset{a}{\mathbf{M}}}$, growth of angular momentum

$\overset{+}{\underset{a}{e}}$, growth of energy

We also assume a system of forces and interactions associated with the detailed structure of the constituents. There are two motivations for this. First, there are the physical arguments of Osborn Reynolds (1885, 1887) indicating that the stress in the voids can be adjusted independently of the stress in the grains. (see also Jenkins, 1975). Second, there is the indication that since the equations are similar in form to other theories with directors, forces the nature of which appear in other theories will also be included here. We shall attempt to deduce physical explanations for these forces.†

†It should be realised that this programme is only partially realised here.

Fields associated with the directors are:

$\underset{a}{k}$, equilibrated inertia

$\underset{a}{\mathbf{h}}$, equilibrated stress

$\underset{a}{l}$, equilibrated body force

$\underset{a}{g}$, intrinsic body force

$\underset{a}{\mathbf{k}}$, equilibrated inertial force

$\underset{a}{K}$, inertial body force

and growth terms are:

$\overset{+}{\underset{a}{v}}$, growth of equilibrated force

$\overset{+}{\underset{a}{k}}$, growth of equilibrated inertia

It is also often convenient to consider the mixture as a single body $\mathcal{B}$. There are no growth terms for $\mathcal{B}$, but otherwise, for each of the terms defined for $\underset{a}{\mathcal{B}}$ there is a similarly named and symbolised quantity for the composite body $\mathcal{B}$; *e.g.* body force $\mathbf{b}$, body heating s. It is reasonable to assume that in some sense, given the properties of the constituents, the properties of the composite body should be determined and, furthermore, the laws of balance for the composite body should be the same as the laws of balance for the constituents. In axiomatic treatments of mixture theory these characteristics are stated precisely, either as axioms or as consequences of the theory. We present them as 'metaphysical principles' in a form often used in mixture theories, and proceed to show that they are satisfied by the theory studied here:†

1. *All properties of the mixture must be mathematical consequences of properties of the constituents.*
2. *So as to describe the motion of a constituent, we may in imagination isolate it from the rest of the mixture, provided we allow properly for the actions of the other constituents upon it.*
3. *The motion of the mixture is governed by the same equations as is a single body.*

Here the 'single body' considered in the third principle is a slight generalisation of the 'granular material' considered by Goodman and Cowin (1972).

11.4. BALANCE PRINCIPLES

We use the symbol Σ henceforth as an abbreviation for summation of a quantity over all n components; thus

$$\sum \equiv \sum_{a=1}^{n}$$

Essentially our plan is to give balance laws for each of the constituents, and then frame the definitions of properties for a composite body in such a fashion that balance laws similar in form hold for that body. This task is not trivial.

The total mass density ρ for the body $\mathscr{B}$ is defined as the sum of the peculiar densities

$$\rho = \sum \underset{a}{\rho} \tag{4.1}$$

and the concentration $\underset{a}{c}$ of the ath constituent is defined by

$$\underset{a}{c} = \underset{a}{\rho}/\rho \tag{4.2}$$

†The principles are quoted from Truesdell (1969, p. 83), where they are subsequently motivated. It may be noted that they are somewhat parallel in intent to the assertion of some molecular theorists that the properties of material bodies are determined by the properties of their molecules.

Let $\mathbf{p}$ be the position vector from some fixed point to $\mathbf{x}$. The densities of linear momentum and angular momentum, as well as equilibrated inertia and volume distribution momentum for $\mathscr{B}$, are defined by

$$\rho\dot{\mathbf{x}} = \sum \underset{a}{\rho}\,\underset{a}{\dot{\mathbf{x}}} \tag{4.3}$$

$$\mathbf{p} \wedge \rho\dot{\mathbf{x}} = \sum \mathbf{p} \wedge \underset{a}{\rho}\,\underset{a}{\dot{\mathbf{x}}} \tag{4.4}$$

and

$$\rho k = \sum \underset{a}{\rho}\,\underset{a}{k} \tag{4.5}$$

$$\rho k \dot{\nu} = \sum \underset{a}{\rho}\,\underset{a}{k}\,\underset{a}{\dot{\nu}} \tag{4.6}$$

Here $\dot{\mathbf{x}}$ is interpreted as velocity in $\mathscr{B}$.

The principle of balance of mass for a constituent is assumed in the form†

$$\underset{a}{\rho}\overset{+}{\underset{a}{c}} = \underset{a}{\dot{\rho}} + \underset{a}{\rho}\,\mathrm{div}\,\underset{a}{\dot{\mathbf{x}}} \equiv \partial_t \underset{a}{\rho} + \mathrm{div}\,(\underset{a}{\rho}\,\underset{a}{\dot{\mathbf{x}}}) \tag{4.7}$$

Let $\underset{a}{\Psi}$ be a scalar, vector or tensor defined at points in $\underset{a}{\mathscr{B}}$. If $\underset{a}{\Psi}$ is related to Ψ by

$$\rho\Psi = \sum \underset{a}{\rho}\,\underset{a}{\Psi} \tag{4.8}$$

then by a standard method it may be shown that

$$\rho\dot{\Psi} = \sum \underset{a}{\rho}\,\underset{a}{\dot{\Psi}} - \mathrm{div} \sum \underset{a}{\rho}\,\underset{a}{\Psi}\,\underset{a}{\mathbf{u}} + \sum \underset{a}{\rho}\,\overset{+}{\underset{a}{c}}\,\underset{a}{\Psi} \tag{4.9}$$

†Here 'div' denotes divergence with respect to $\mathbf{x}$. The divergence of a vector is the trace of its gradient. Gradient is denoted by 'grad', trace by 'tr'.

where $\underset{a}{\mathbf{u}}$ is the diffusion velocity of the ath constituent:

$$\underset{a}{\mathbf{u}} = \underset{a}{\dot{\mathbf{x}}} - \dot{\mathbf{x}} \tag{4.10}$$

By (4.9), (4.3)–(4.6) become†

$$\rho\ddot{\mathbf{x}} = \sum \underset{a}{\rho}\underset{a}{\ddot{\mathbf{x}}} - \operatorname{div}\sum \underset{a}{\rho}\underset{a}{\dot{\mathbf{x}}}\times\underset{a}{\mathbf{u}} + \sum \underset{a}{\rho}\overset{+}{c}\underset{a}{\dot{\mathbf{x}}} \tag{4.11}$$

$$\rho\dot{k} = \sum \underset{a}{\rho}\underset{a}{\dot{k}} - \operatorname{div}\sum \underset{a}{\rho}\underset{a}{k}\underset{a}{\mathbf{u}} + \sum \underset{a}{\rho}\overset{+}{c}\underset{a}{k} \tag{4.12}$$

$$\rho k\ddot{\nu} + \rho\dot{k}\dot{\nu} = \sum \underset{a}{\rho}(\underset{a}{k}\underset{a}{\ddot{\nu}} + \underset{a}{\dot{k}}\underset{a}{\dot{\nu}}) - \operatorname{div}\sum \underset{a}{\rho}\underset{a}{k}\underset{a}{\dot{\nu}}\underset{a}{\mathbf{u}} + \sum \underset{a}{\rho}\overset{+}{c}\underset{a}{k}\underset{a}{\dot{\nu}} \tag{4.13}$$

Thus, for instance, the definition of velocity for the mixture from properties of the constituents implies the definition of acceleration for the mixture in terms of properties of the constituents.

Balance laws for momentum, moment of momentum, energy, equilibrated force and equilibrated inertia are given by

$$\rho\underset{a}{\overset{+}{\mathbf{m}}} = \underset{a}{\rho}\underset{a}{\ddot{\mathbf{x}}} + \underset{a}{\rho}\underset{a}{\dot{\mathbf{x}}}\overset{+}{c} - \operatorname{div}\underset{a}{\mathbf{T}} - \underset{a}{\rho}\underset{a}{\mathbf{b}} \tag{4.14}$$

$$\begin{aligned}\rho\underset{a}{\overset{+}{\mathbf{M}}} + \mathbf{p}\wedge\rho\underset{a}{\overset{+}{\mathbf{m}}} &= \mathbf{p}\wedge\underset{a}{\rho}\underset{a}{\ddot{\mathbf{x}}} + \mathbf{p}\wedge\underset{a}{\rho}\underset{a}{\dot{\mathbf{x}}}\overset{+}{c} - \mathbf{p}\wedge\operatorname{div}\underset{a}{\mathbf{T}}\\ &\quad + \underset{a}{\mathbf{T}} - \underset{a}{\mathbf{T}}^T - \mathbf{p}\wedge\underset{a}{\rho}\underset{a}{\mathbf{b}}\end{aligned} \tag{4.15}$$

$$\begin{aligned}\rho\underset{a}{\overset{+}{e}} &= \underset{a}{\rho}(\underset{a}{\dot{\epsilon}} + \underset{a}{\dot{\mathbf{x}}}\,.\,\underset{a}{\ddot{\mathbf{x}}} + \underset{a}{k}\underset{a}{\dot{\nu}}\underset{a}{\ddot{\nu}} + \tfrac{1}{2}\underset{a}{\dot{k}}\underset{a}{\dot{\nu}}^2) + \underset{a}{\rho}\overset{+}{c}(\underset{a}{\epsilon} + \tfrac{1}{2}\underset{a}{\dot{x}}^2 + \tfrac{1}{2}\underset{a}{k}\underset{a}{\dot{\nu}}^2) - \underset{a}{\dot{\mathbf{x}}}\,.\operatorname{div}\underset{a}{\mathbf{T}}\\ &\quad - \operatorname{tr}(\underset{a}{\mathbf{T}}^T\operatorname{grad}\underset{a}{\dot{\mathbf{x}}}) - \operatorname{div}\underset{a}{\mathbf{q}} - \underset{a}{\dot{\nu}}\operatorname{div}\underset{a}{\mathbf{h}} - \underset{a}{\mathbf{h}}\,.\operatorname{grad}\underset{a}{\dot{\nu}} - \underset{a}{\rho}(\underset{a}{s} + \underset{a}{\dot{\mathbf{x}}}\,.\,\underset{a}{\mathbf{b}} + \underset{a}{\dot{\nu}}\underset{a}{l})\end{aligned} \tag{4.16}$$

†Details of these, and most other calcuiations in this work, are given by Passman (1974a).

$$\rho\underset{a}{\overset{+}{v}} = \underset{a}{\rho}\underset{a}{k}\underset{a}{\ddot{\nu}} + \underset{a}{\rho}\underset{a}{\dot{k}}\underset{a}{\dot{\nu}} + \underset{a}{\rho}\underset{a}{k}\underset{a}{\dot{\nu}}\overset{+}{c} - \operatorname{div}\underset{a}{\mathbf{h}} - \underset{a}{\rho}(\underset{a}{l} + \underset{a}{g}) \tag{4.17}$$

$$\rho\underset{a}{\overset{+}{k}} = \underset{a}{\rho}\underset{a}{\dot{k}} + \underset{a}{\rho}\overset{+}{c}\underset{a}{k} - \operatorname{div}\underset{a}{\mathbf{k}} - \underset{a}{\rho}\underset{a}{K} \tag{4.18}$$

By (4.7), these are equivalent to the set

$$\rho\underset{a}{\overset{+}{c}} = \underset{a}{\dot{\rho}} + \underset{a}{\rho}\operatorname{div}\underset{a}{\dot{\mathbf{x}}} \tag{4.7}$$

$$\rho\underset{a}{\overset{+}{\mathbf{m}}} = \underset{a}{\rho}\underset{a}{\ddot{\mathbf{x}}} + \underset{a}{\rho}\underset{a}{\dot{\mathbf{x}}}\overset{+}{c} - \operatorname{div}\underset{a}{\mathbf{T}} - \underset{a}{\rho}\underset{a}{\mathbf{b}} \tag{4.19}$$

$$\rho\underset{a}{\overset{+}{\mathbf{M}}} = \underset{a}{\mathbf{T}} - \underset{a}{\mathbf{T}}^T \tag{4.20}$$

$$\begin{aligned}\rho\underset{a}{\overset{+}{e}} &= \underset{a}{\rho}\underset{a}{\dot{\epsilon}} + \underset{a}{\rho}\overset{+}{c}(\underset{a}{\epsilon} - \tfrac{1}{2}\underset{a}{\dot{x}}^2) + \rho\underset{a}{\overset{+}{\mathbf{m}}}\,.\,\underset{a}{\dot{\mathbf{x}}} + \rho\underset{a}{\overset{+}{v}}\underset{a}{\dot{\nu}} - \rho\underset{a}{\overset{+}{k}}(\tfrac{1}{2}\underset{a}{\dot{\nu}}^2)\\ &\quad + \underset{a}{\rho}\underset{a}{g}\underset{a}{\dot{\nu}} - \operatorname{tr}(\underset{a}{\mathbf{T}}^T\operatorname{grad}\underset{a}{\dot{\mathbf{x}}}) - \operatorname{div}\underset{a}{\mathbf{q}} - \underset{a}{\mathbf{h}}\,.\operatorname{grad}\underset{a}{\dot{\nu}} - \underset{a}{\rho}\underset{a}{s}\\ &\quad - (\underset{a}{\rho}\underset{a}{K} + \operatorname{div}\underset{a}{\mathbf{k}})\tfrac{1}{2}\underset{a}{\dot{\nu}}^2\end{aligned} \tag{4.21}$$

$$\rho\underset{a}{\overset{+}{v}} = \underset{a}{\rho}\underset{a}{k}\underset{a}{\ddot{\nu}} + \underset{a}{\rho}\underset{a}{\dot{k}}\underset{a}{\dot{\nu}} + \underset{a}{\rho}\underset{a}{k}\underset{a}{\dot{\nu}}\overset{+}{c} - \operatorname{div}\underset{a}{\mathbf{h}} - \underset{a}{\rho}(\underset{a}{l} + \underset{a}{g}) \tag{4.22}$$

$$\rho\underset{a}{\overset{+}{k}} = \underset{a}{\rho}\underset{a}{\dot{k}} + \underset{a}{\rho}\overset{+}{c}\underset{a}{k} - \operatorname{div}\underset{a}{\mathbf{k}} - \underset{a}{\rho}\underset{a}{K} \tag{4.23}$$

Equations (4.7) and (4.19)–(4.23) appear to satisfy the second metaphysical principle.

We define quantities for the composite body in terms of quantities for the components according to the following equations:

$$
\begin{aligned}
\mathbf{T} &= \sum \underset{a}{\mathbf{T}} - \sum \underset{a}{\rho}\underset{a}{\mathbf{u}} \times \underset{a}{\mathbf{u}} \\
\mathbf{q} &= \sum \underset{a}{\mathbf{q}} + \sum [\underset{a}{\mathbf{T}}^T \underset{a}{\mathbf{u}} + \underset{a}{\mathbf{h}}(\underset{a}{\dot{\nu}} - \dot{\nu}) - \underset{a}{\rho}(\underset{a}{\epsilon} + \tfrac{1}{2}\underset{a}{u}^2 - \underset{a}{k}\underset{a}{\dot{\nu}}(\underset{a}{\dot{\nu}} - \tfrac{1}{2}\dot{\nu}))\underset{a}{\mathbf{u}}] \\
\mathbf{h} &= \sum \underset{a}{\mathbf{h}} - \sum \underset{a}{\rho}\underset{a}{k}\underset{a}{\dot{\nu}}\underset{a}{\mathbf{u}}, \quad \mathbf{k} = \sum \underset{a}{\mathbf{k}} - \sum \underset{a}{\rho}\underset{a}{k}\underset{a}{\mathbf{u}}, \quad \rho\mathbf{b} = \sum \underset{a}{\rho}\underset{a}{\mathbf{b}} \\
\rho s &= \sum \underset{a}{\rho}\underset{a}{s} + \sum \underset{a}{\rho}[\underset{a}{\mathbf{b}} \cdot \underset{a}{\mathbf{u}} + \underset{a}{l}(\underset{a}{\dot{\nu}} - \dot{\nu})] \\
\rho\epsilon &= \sum \underset{a}{\rho}\underset{a}{\epsilon} + \sum \tfrac{1}{2}\underset{a}{\rho}[\underset{a}{u}^2 + \underset{a}{k}(\underset{a}{\dot{\nu}} - \dot{\nu})^2] \\
\rho l &= \sum \underset{a}{\rho}\underset{a}{l}, \quad \rho g = \sum \underset{a}{\rho}\underset{a}{g}, \quad \rho K = \sum \underset{a}{\rho}\underset{a}{K}
\end{aligned}
\tag{4.24}
$$

These definitions, along with (4.3)–(4.6), appear to satisfy the first metaphysical principle.

Assume that mass, linear momentum, angular momentum, energy, equilibrated force and equilibrated inertia are each conserved for the mixture, *i.e.*

$$
\begin{aligned}
&\sum \overset{+}{\underset{a}{c}} = 0, \quad \sum \overset{+}{\underset{a}{\mathbf{m}}} = 0, \quad \sum \overset{+}{\underset{a}{\mathbf{M}}} = 0, \\
&\sum \overset{+}{\underset{a}{e}} = 0, \quad \sum \overset{+}{\underset{a}{v}} = 0, \quad \sum \overset{+}{\underset{a}{k}} = 0
\end{aligned}
\tag{4.25}
$$

A routine, but quite tedious, set of manipulations then leads to the equations of balance:

$$
\begin{gathered}
\dot{\rho} + \rho \operatorname{div} \dot{\mathbf{x}} = 0 \\
\rho\ddot{\mathbf{x}} = \rho\mathbf{b} + \operatorname{div}\mathbf{T} \\
\mathbf{T} = \mathbf{T}^T \\
\rho\dot{\epsilon} = \operatorname{tr}(\mathbf{T}^T \operatorname{grad}\dot{\mathbf{x}}) + \mathbf{h} \cdot \operatorname{grad}\dot{\nu} + \tfrac{1}{2}\rho\dot{k}\dot{\nu}^2 + \rho g\dot{\nu} + \operatorname{div}\mathbf{q} + \rho s
\end{gathered}
$$

$$
\begin{gathered}
\rho k\ddot{\nu} + \rho\dot{k}\dot{\nu} = \operatorname{div}\mathbf{h} + \rho(l + g) \\
\rho\dot{k} - \operatorname{div}\mathbf{k} - \rho K = 0
\end{gathered}
\tag{4.26}
$$

for $\mathscr{B}$. Conversely, if (4.26) hold, then (4.25) follow. The set of equations given above is too general to yield a decision as to whether the third metaphysical principle is satisfied. In the theory of Goodman and Cowin (1972):

$$
\operatorname{div}\mathbf{k} = 0, \quad K = 0 \tag{4.27}
$$

and $(4.26)_{3,4,5}$ become

$$
\begin{gathered}
\rho\dot{\epsilon} = \operatorname{tr}(\mathbf{T}^T \operatorname{grad}\dot{\mathbf{x}}) + \mathbf{h} \cdot \operatorname{grad}\dot{\nu} + \rho g\dot{\nu} + \operatorname{div}\mathbf{q} + \rho s \\
\rho k\ddot{\nu} = \operatorname{div}\mathbf{h} + \rho(l + g), \quad \dot{k} = 0
\end{gathered}
\tag{4.28}
$$

The set $(4.26)_{1,2,3}$ are the usual balance laws for mass, linear and angular momentum for a continuum. Equation $(4.28)_1$ is the balance of energy for a granular medium as given by Goodman and Cowin (1972). It differs from the usual balance of energy only through the inclusion of power terms due to equilibrated stress and intrinsic body force. Equations $(4.28)_{2,3}$ for equilibrated force and equilibrated inertia are the same as those of Goodman and Cowin. They do not appear in the theory of non-polar continua.

Some indication of the meaning of the balance laws for equilibrated force may be derived from the following argument. Consider the balance law $(4.28)_2$. Recall that ν is a measure of the ratio of volume occupied by grains. Consider a given volume, and assume it contains a fixed number of grains, each spherical and uniform in size. In this case, an example of such a measure is the radius of a typical grain. In effect, this is a reinterpretation of ν, not as a volume ratio but as a measure of typical radius. Consider $\mathbf{h}$ to be proportional to the force vector acting on a point on the boundary of the grain, and $l + g$ to be proportional to the pressure on the interior of a grain. Then the balance law for equilibrated force is an inertial law for the radii of grains. In the case where the grains consisted of a perfectly elastic material subject to infinitesimal deformation, the equilibrated inertia k would be proportional to $\lambda + 2\mu$, where λ and μ are the usual Lamé constants.

11.5. THE ENTROPY PRINCIPLE

In works on thermodynamics, it is common to introduce a concept of temperature and a concept of entropy, and then impose a growth law for the entropy, sometimes called the Second Law of Thermodynamics.

We consider the ath component of the mixture, and call its (specific) entropy $\underset{a}{\eta}$, its entropy growth $\overset{+}{\underset{a}{\eta}}$, and its entropy supply s. We assume the existence of an absolute temperature θ, $\theta > 0$, which can vary in space and time, but which is the same for each component of the mixture. For convenience we introduce the coldness ϑ as the reciprocal of the temperature:

$$\vartheta = 1/\theta \tag{5.1}$$

We postulate the balance law for the entropy of each constituent:

$$\underset{a}{\rho}\overset{+}{\eta} = \underset{a}{\rho}\underset{a}{\eta}^{\cdot} + \underset{a}{\rho}\overset{+}{\underset{a}{c}}\eta - \operatorname{div}(\vartheta \underset{a}{\mathbf{q}}) - \vartheta \underset{a}{\rho}\underset{a}{s} \tag{5.2}$$

Define the total entropy η, the entropy flux φ, and the entropy supply s by

$$\rho\eta = \sum \underset{a}{\rho}\underset{a}{\eta} \tag{5.3}$$

$$\varphi = \sum (\vartheta \underset{a}{\mathbf{q}} - \underset{a}{\rho}\underset{a}{\eta}\underset{a}{\mathbf{u}}) \tag{5.4}$$

$$\rho s = \sum \underset{a}{\rho}\underset{a}{s} \tag{5.5}$$

We assume the axiom of dissipation:

$$\sum \overset{+}{\underset{a}{\eta}} \geqslant 0 \tag{5.6}$$

which holds if and only if

$$\rho\dot{\eta} \geqslant \operatorname{div} \varphi + \rho\vartheta s \tag{5.7}$$

This result is the form of the entropy inequality to be exploited in this work.

We introduce the free energy ψ:

$$\psi = \epsilon - \eta/\vartheta \tag{5.8}$$

the following kinematical quantities:

$\mathbf{G} = \operatorname{grad} \dot{\mathbf{x}}$, the velocity gradient

$\underset{a}{\mathbf{G}} = \operatorname{grad} \underset{a}{\dot{\mathbf{x}}}$, the peculiar velocity gradients

$\mathbf{D}$, the symmetric part of $\mathbf{G}$, called the stretching (5.9)

$\mathbf{W}$, the skew part of $\mathbf{G}$, called the spin, and similarly $\underset{a}{\mathbf{D}}$ and $\underset{a}{\mathbf{W}}$

and the quantity

$$\hat{\varphi} = \varphi - \vartheta\mathbf{q} + \vartheta \sum \underset{a}{\mathbf{T}}^{T} \underset{a}{\mathbf{u}} + \vartheta \sum \underset{a}{\mathbf{h}}(\underset{a}{\dot{\nu}} - \dot{\nu}) \tag{5.10}$$

A routine but quite tedious set of calculations then leads to the following form for the entropy inequality:

$$\begin{aligned}
\rho\eta\frac{\dot{\vartheta}}{\vartheta} - \rho\vartheta\dot{\psi} \geqslant{}& \operatorname{div}\hat{\varphi} - \vartheta\Big[\sum \underset{a}{\rho}\underset{a}{\ddot{\mathbf{x}}} \,.\, \underset{a}{\mathbf{u}} + \sum (\underset{a}{\rho}\underset{a}{k}\underset{a}{\ddot{\nu}} + \underset{a}{\rho}\underset{a}{\dot{k}}\underset{a}{\dot{\nu}})(\underset{a}{\dot{\nu}} - \dot{\nu})\Big] \\
&+ \Big[\mathbf{q} - \sum \underset{a}{\mathbf{T}}^{T}\underset{a}{\mathbf{u}} - \sum \underset{a}{\mathbf{h}}(\underset{a}{\dot{\nu}} - \dot{\nu})\Big] \,.\, \operatorname{grad}\vartheta \\
&+ \vartheta\rho \sum (\overset{+}{\underset{a}{\mathbf{m}}} - \overset{+}{\underset{a}{c}}\underset{a}{\dot{x}}) \,.\, \underset{a}{\mathbf{u}} + \vartheta\rho \sum (\overset{+}{\underset{a}{v}} - \overset{+}{\underset{a}{c}}\underset{a}{k}\underset{a}{\dot{\nu}})(\underset{a}{\dot{\nu}} - \dot{\nu}) \\
&- \frac{\vartheta}{\rho}\operatorname{tr}\Big(\mathbf{T} - \sum \underset{a}{\mathbf{T}}\Big)\Big(\sum \underset{a}{\rho}\underset{a}{\mathbf{D}} + \sum \underset{a}{\mathbf{u}} \times \operatorname{grad} \underset{a}{\rho}\Big) - \vartheta \sum \operatorname{tr} \underset{a}{\mathbf{T}}^{T}\underset{a}{\mathbf{G}}
\end{aligned}$$

$$-\frac{\vartheta}{\rho k}\Big(\mathbf{h}-\sum \underset{a}{\mathbf{h}}\Big)\cdot\Big[\sum \underset{aa}{\rho k}\ \mathrm{grad}\ \underset{a}{\dot\nu}+\sum(\underset{a}{\dot\nu}-\dot\nu)(\mathrm{grad}\ \underset{aa}{\rho k})\Big]$$

$$-\vartheta\sum \underset{a}{\mathbf{h}}\ .\ \mathrm{grad}\ \underset{a}{\dot\nu}-\vartheta\Big(\rho g\dot\nu-\sum \underset{aa}{\rho g}(\underset{a}{\dot\nu}-\dot\nu)+\tfrac{1}{2}\rho\dot{k}\dot\nu^{2}\Big) \qquad (5.11)$$

ACKNOWLEDGEMENTS

I wish to express my appreciation to the many individuals who unselfishly gave of their time and effort in order to comment on this work. These include, in addition to almost all the authors of the cited papers, Professors J. L. Ericksen and K. Walters. My special thanks also go to Dr P. A. C. Raats, who introduced me to soil mechanics. The research was initiated under grants to the Soil Science Department and Mathematics Research Center of the University of Wisconsin, and further supported by grants from the U.S. National Science Foundation to the Georgia Institute of Technology.

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E-23-606

GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA 30332

OFFICE OF
THE DIRECTOR OF
FINANCIAL AFFAIRS

December 28, 1976

Grants & Contracts Office
National Science Foundation
Washington, D. C. 20550

Gentlemen:

Enclosed is the original and two copies of the final fiscal report for grant number ENG73-03938 A02 (formerly GK-40181). The final technical report was mailed on December 6, 1976.

If you have any questions or desire additional information, please let us know.

Sincerely yours,

E. Evan Crosby
Associate Director of
Financial Affairs

ECE/bs
enclosure:
cc: Dr. S. L. Passman w/copy
Mr. E. E. Renfro w/copy
Mr. A. H. Becker w/copy
File E-23-606

AN EQUAL EDUCATION AND EMPLOYMENT OPPORTUNITY INSTITUTION

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

RESEARCH GRANT BUDGET & FISCAL REPORT

Form Approved
Budget Bureau No. 99-R0013

Please read instructions on reverse side carefully before completing this form.

INSTITUTION AND ADDRESS		NSF PROGRAM	GRANT PERIOD
Georgia Institute of Technology Atlanta, Georgia		Solid Mechanics	from 10/1/73 to 3/31/76
			REPORTING PERIOD from 10/1/73 to 12/16/76*
GRANT NUMBER ENG73-03938 A02	BUDGET DUR. (MOS.) 24	PRINCIPAL INVESTIGATOR(S) Passman	GRANTEE ACCOUNT NUMBER E-23-606

	NSF Funded Man Months Cal.	Acad.	Summ.	NSF AWARD BUDGET	CUMULATIVE GRANT EXPENDITURES *Do Not Round*
A. SALARIES AND WAGES					
1. Senior Personnel					
a. 1 (Co)Principal Investigator(s)		3	2	$ 11,536	
b. Faculty Associates					
Sub-Total				$ 11,536	$ 17,536.64
2. Other Personnel (Non-Faculty)					
a. Research Associates—Postdoctoral					
b. Non-Faculty Professionals					
c. 1 Graduate Students				6,400	
d. Pre-Baccalaureate Students					
e. Secretarial—Clerical					
f. Technical, Shop, and Other					
TOTAL SALARIES AND WAGES				$ 17,936	$ 17,536.64
B. STAFF BENEFITS IF CHARGED AS DIRECT COST				1,525	1,551.22
C. TOTAL SALARIES, WAGES, AND STAFF BENEFITS (A + B)				$ 19,461	$ 19,087.86
D. PERMANENT EQUIPMENT					-0-
E. EXPENDABLE EQUIPMENT AND SUPPLIES					-0-
F. TRAVEL 1. DOMESTIC (INCLUDING CANADA)				500	195.73
2. FOREIGN					-0-
G. PUBLICATION COSTS				500	9.00
H. COMPUTER COSTS IF CHARGED AS DIRECT COST					-0-
I. OTHER DIRECT COSTS					-0-
J. TOTAL DIRECT COSTS (C through I)				$ 20,461	$ 19,292.59
K. INDIRECT COSTS 65% of Salaries and Wages				11,658	** 11,665.67
L. TOTAL COSTS (J plus K)				$ 32,119	$ 30,958.26
M. AMOUNT OF THIS AWARD (ROUNDED)				$ 32,100	
N. CUMULATIVE GRANT AMOUNT				$	
O. UNEXPENDED BALANCE (N. BUDGET MINUS L. EXPENDITURE)					$ 1,141.74

REMARKS: Use extra sheet if necessary.

**65% of $8,641.64 = $ 5,617.07
68% of $8,895.00 = $ 6,048.60
$11,665.67

*No obligations were incurred outside the grant period of 10/1/73 through 8/31/76.

FOR NSF USE ONLY
Final Fiscal Report Accepted
Grant Closed ____ Remains Open ____
By ________ Date ________
Grants Administration Section, Area ____

SIGNATURE OF PRINCIPAL INVESTIGATOR	TYPED OR PRINTED NAME	DATE
[signature]	S. L. Passman	12/27/76

I CERTIFY THAT ALL EXPENDITURES REPORTED ARE FOR APPROPRIATE PURPOSES AND IN ACCORDANCE WITH THE AGREEMENTS SET FORTH IN THE APPLICATION AND AWARD DOCUMENTS

SIGNATURE OF AUTHORIZED OFFICIAL	TYPED OR PRINTED NAME & TITLE	DATE
[signature]	C. Evan Crosby, Associate Director of Financial Affairs	12/28/76

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Organ. Code	F.Y.	Fund ID	Prog. Code	Ob. Class	O/Dres.	Award No.	Amd.	Inst. Code	Unexpended Balance	Trans.	Lot
									$		

E-25-608

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

November 30, 1976

Dr. Clifford Astill
Solid Mechanics Program
National Science Foundation
Washington, D.C. 20550

Dear Dr. Astill:

The following information is presented as the final report on N.S.F. Grant Number GK-40181, "Solutions in Cosserat Surface Theory."

As has been explained in the progress reports of 1974 and 1975, the research has developed in such a fashion that it appeals to a broader set of theories of Cosserat continua than Cosserat surfaces. However, early in the progress of the research a rather deep question came up which appears to have ramifications of particular significance in Cosserat surface theory (and therefore shell theory). A substantial portion of time has been utilized in investigating the question.

A paper [1], "An Exact Solution in Cosserat Surface Theory" was presented at the Eighth Southeastern Conference on Theoretical and Applied Mechanics.

Shortly after beginning this research project, I received a manuscript on the theory of inertia by S. C. Cowin and F. Leslie. At the time it appeared to me that it would be a straight forward job to extend their ideas to the case of Cosserat surfaces and that it might shed some light on the concept of "rotary inertia" in shell theory. After some progress in doing so, I lectured on this subject at the Twelfth Meeting of the Society of Engineering Science in 1975 [2]. At that meeting, Professor J. G. Simmonds pointed out to me that he had postulated in his shell theory inertial terms of the form which are derived in my presentation.

Some reflection on the theory of inertia made clear to me the fact that, although Cowin and Leslie's theory presents necessary conditions for an inertial theory, some new physical principle is needed to provide sufficient conditions leading to the "right" answers. This general idea was presented at invited lectures at Tulane University (February 1976), Cornell University (April 1976), and Georgia Tech (May 1976). Since then, I feel I have satisfactorily formulated the required principle and a paper on this subject

[3] is now being typed. A paper on its application to Cosserat surface theory [4] is being written out.

Substantial effort has also been expended in developing Cosserat models for soils. The basic model, that for a dry granular medium, has been developed by Goodman and Cowin. Extension of this model to the case of a mixture of such media, possibly including partially fluid-filled interstices and non-contact interparticle interactions could serve as a model for some soils in nature. My research effort toward this end begun before the inception of the grant presently being reported upon, and is expected to continue for some time. Only that work done during the duration of this project is reported upon here. The basic laws of balance for such continua have been presented at the conference, "Theoretical Rheology," at Cambridge, England (September 1974) and published in the proceedings of that conference [5]. Constitutive theories were lectured upon at the Italian-American Conference on Mixtures and Oriented Materials in Udine, Italy (June 1974) and at the University of Ireland (September 1974). A paper on this subject is now in press [6] but much work remains to be done. A graduate student, Mr. John P. Thomas, Jr., also worked on the theory of a single granular medium, and found a number of exact solutions for flow, some of which are universal, but not viscometric, motions. His thesis on this topic has been completed [7], and publication is planned.

A list of publications and talks given under the auspices of this grant, copies of papers for which reprints are available, and Forms 98 and 98A are appended.

The author expresses his sincere appreciation for having been able to work on this research project.

Respectfully,

S. L. Passman
Associate Professor

/pg
Enclosure

Papers Published (copies enclosed for those marked with an asterisk.)

[1]* An exact solution in Cosserat surface theory, Developments in Theoretical and Applied Mechanics VIII, 23-30, edited by R. P. McNitt, Blacksburg (1976).

[2] The kinetic energy of a Cosserat surface, Proceedings, 12th Annual Meeting of the Society of Engineering Science, 1051, (abstract only) edited by M. Stern, Austin (1975).

[3] On the theory of energy and momentum, currently being typed (approximately 25 pages).

[4] The kinetic energy of a Cosserat surface, currently being written out for publication.

[5]* Balance laws for mixtures of granular materials, Theoretical Rheology, 169-185, edited by J. F. Hutton, J. R. A. Pearson, and K. Walters, Barking, England (1975).

[6]* Mixtures of granular materials, International Journal of Engineering Science, (13 pages), to appear (1977).

[7]* Viscometric motions of granular materials, by J. P. Thomas, Jr., M.S.E.M. thesis, Georgia Institute of Technology, 118 pages (1976).

[8] d'Alembert flows of viscous fluids, by John P. Thomas, Jr., currently being typed.

Lectures Presented (Invited lectures marked with an asterisk. Corresponding paper indicated.)

*Italian-American Conference on Mixtures and Oriented Materials, Udine, Italy, June 1974 [6].

*University of Ireland, Galway, September 1974 [6].

*Conference on Theoretical Rheology, Cambridge, England, September 1974 [5].

*Society of Engineering Science, Austin, October 1975 [2].

*Tulane University, February 1976 [4].

*Cornell University, April 1976 [4].

Eighth Southeastern Conference on Theoretical and Applied Mechanics, Blacksburg, April 1976 [1].

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

SUMMARY OF COMPLETED PROJECT

Form Approved
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Please read instructions on reverse carefully before completing this form.

1. INSTITUTION AND ADDRESS	2. NSF PROGRAM	3. GRANT PERIOD from to

4. GRANT NUMBER	5. BUDGET DUR. (MOS)	6. PRINCIPAL INVESTIGATOR(S)	7. GRANTEE ACCOUNT NUMBER

8. SUMMARY (Attach list of publications to form)

The mathematical modeling of the behavior of continuous media is a necessary explicit step in the solution of engineering problems. This step is often quite easy because a standard model, such as that of a non-polar ordinary continuum, is appropriate. Increasingly, however, the advent of exotic materials, renewed interest in natural materials, and the need to analyze complex structures have made it necessary for the engineer to pay greater attention to the mathematical model appropriate to the material or structure he uses.

This project considers a set of models called "Cosserat materials" or "Cosserat continua". Such materials are more general than ordinary continuous media and include, e.g., materials with couple stresses, some biological fluids, several types of soils and rod and shell-like structures.

Shell-like structures are of considerable utility in engineering practice. The usual derivation of their governing equations, however, is so complex, especially so for the case of large deformations, that the possibility of solving problems with them, except possibly by numerical techniques, is rather meager. In a paper written previous to the inception of this project, the author showed that the equations isomorphic to those of shell theory, the "Cosserat shell", could be derived in a straightforward fashion. Although not the first derivation of these equations, the paper presents a direct notation which emphasizes their simple structure, as opposed to previous works which emphasize their algebraic complexity.

As originally conceived, this research was intended as a study of the equations of Cosserat surface theory, and a first paper [1], was written on that topic. In the process of working out that paper, it became obvious that, although their interpretations are widely varied, the basic mathematical structures of all Cosserat theories are the same, and often results obtained in the context of one theory may be "read off" in the context of other theories. It is therefore possible to direct attention to whatever theory seems to present the most challenging problem, with the assurance that its solution may be applied, with due caution, to another theory.

One problem of interest in the context of the Cosserat shell is that, although the equations of statics are well-understood, the equations of dynamics were still open to debate. There are papers on this question in the literature, but the choices of inertial terms are usually ad hoc. In fact, a close look indicates that there are many possible forms of inertia, all consistent with known principles of mechanics. What was obviously necessary was a theory of inertia. Such a theory had been previously initiated by other authors, but it needed completion before it could be applied to this case. Substantial effort was put into this matter, and it has now been satisfactorily resolved. A paper on this subject [3] is forthcoming. A further paper applying it to Cosserat surface theory [4], based on a lecture given in 1975 [2], is now being written out.

An interesting application of a Cosserat theory is to the theory of granular media. Several boundary-value problems relevant to a dry granular medium were solved by Mr. J. P. Thomas [7], a student of the Principal Investigator. The investigator himself has placed some effort into extending the model to cases possibly appropriate to some soils found in nature. The mathematical complexity of such models is formidable,

9. SIGNATURE OF PRINCIPAL INVESTIGATOR/ PROJECT DIRECTOR	TYPED OR PRINTED NAME	DATE

NSF Form 98A (5-76)

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SUMMARY OF COMPLETED PROJECT

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OMB No. 99R0013

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4. GRANT NUMBER	5. BUDGET DUR. (MOS)	6. PRINCIPAL INVESTIGATOR(S)	7. GRANTEE ACCOUNT NUMBER

8. SUMMARY (Attach list of publications to form)

reflecting the possible variety of interactions in soils. Nonetheless, substantial progress has been made. Fully general equations of balance were given [5] in an early stage of the research. A set of constitutive equations of limited generality has been constructed [6]. Further research on this topic is planned for the future.

9. SIGNATURE OF PRINCIPAL INVESTIGATOR/ PROJECT DIRECTOR	TYPED OR PRINTED NAME	DATE

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